Chapter 2: Prices of Futures Contracts

Problem 1: Suppose the current futures price for delivery of gold in 60 days is $375 per troy oz. The current spot price of gold is $365 per troy oz. Storing gold costs $12 per oz. per year, and the storage costs are paid when the gold is taken out of storage. The short-term interest rate is 6% per year. (We can borrow or lend at this rate.) Do all of the following calculations based on a 360-day year.

a. What interest rate can we earn using a synthetic lending strategy?

If we engage in a cash-and-carry synthetic lending operation, we will:

I. Buy spot gold at $365 per oz.

II. Go short the gold futures contract, expiring 60 days from now, which locks in a price of $375/oz.

III. Pay a storage fee of ($60/360)($12) = $2 per oz. [note here that we reduce the annual cost to 2

Months the period over which the contract runs]

So, our rate of return will be:

= ($375 - $2 - $365) / $365 = 2.19% for 60 days

On an annualized basis your return will be: 2.19% x 6 [2 month periods in a year] = 13.14%

b. Is there an arbitrage opportunity? If so, how would you pursue it?

There is an arbitrage opportunity because you can borrow at 6% and lend at 13.14%. Thus, we should borrow money, buy gold in the spot market, go short futures, and complete a cash-and-carry strategy, with an implied repo rate of: 13.14% - 6% or 7.14% on an annual basis.

c. What will happen to spot and futures prices of gold as the market participants pursue arbitrage?

As arbitrage continues to develop, the spot price of gold will rise as more participants seek to buy gold at $365, at the same time the gold futures contract will fall due to the selling of these contracts beginning at $375. When the implied repo rate reaches 0 there will be no further arbitrage possibilities and both markets will be in equilibrium.

d. What do the above calculations tell us about how we would expect the futures to be priced in equilibrium. The futures price should adjust to the spot price including borrowing costs along with storage costs built in.

$F_{t,T} = P_t (1 + r_{t,T}) + FV$ (Storage)

$= \$365(1 + 60/360 (.06)) + \$2 = \$370.65$

Notice that at this price the implied repo rate would be:

Implied Repo Rate = $(\$370.65 - \$2 - \$365)/365 = 1\%$ for 60 days,

which implies: $1\% \times 6 = 6\%$ for 1 year.

2. Suppose the S&P 1 index is made up solely of IBM stock [1 share]. You observe the following data on March 15th:

IBM stock	$125
September 15 S&P 1 futures price	$123
Borrowing and Lending Rate	7% per annum
Dividend in IBM (paid the first day of February, May, August and November)	$2.50

a. Construct a strategy to synthetically borrow (risklessly) between March 15 and September 15.

To perform a synthetic borrowing hedge you will perform steps just the opposite from a synthetic lending arbitrage strategy:

I. Sell IBM stock short, which will bring in $125 on March 15th

II. Go long the S&P 1 futures contract. This will lock in a purchase price of the IBM stock

In a purchase price on the stock of $123 September 15th permitting you to cover your short.

III. Borrow to pay the dividends to the party from whom you borrowed the stock. Thus, you will

Borrow $2.50 on May 1 and August 1. On September 15th you must pay back the loan plus

Interest.

Cost of Dividends:

Cost for May Dividend == \$2.50 (1 + (137/360) (.07)) = \$2.57

Cost for August Dividend = \$2.50(1 + (45/360)(.07)) = \$2.52

Therefore, the rate that you pay between March 15^{th} and September 15^{th} is:

Rate Paid = (\$123 + \$2.57 + \$2.52 - \$125)/\$125 = 2.47% for 184 days

======➔ 360/184 x 2.47% or 4.83% for 360 days

Because this is a synthetic borrowing rate, it is an implied reverse repo rate.

c. How does the synthetic borrowing rate compare with the actual borrowing rate? Does this indicate there is an arbitrage opportunity? If so, is this a cash-and-carry or a reverse cash-and carry arbitrage opportunity?

In this instance, you are looking at a synthetic borrowing arbitrage and so this would involve a reverse cash-and-carry strategy. Since the implied reverse r epo rate is 4.83% < the lending rate of 7%, there is a reverse cash-and-carry opportunity.

d. Notice that the September 15 S&P 1 futures price is below the spot price of IBM stock on March 15. Does this immediately imply that an arbitrage opportunity exists?

Not necessarily, the futures price can be below the spot price, because the dividend yield may be greater than the short-term interest rate.

e. What is the no-arbitrage price for the futures contract? Relate you answer here to that in part c.

The no-arbitrage price will be:

Futures Price = $P_t\ (1 + r_{t,T})$ - FV(Dividends from t to T)

= \$125(1 + (184/360)(.07)) – (\$2.57 + \$2.52) = \$124.38

Note that \$124.38 > the actual futures price of \$123, so that a reverse cash and carry arbitrage opportunity exists, a finding consistent with c.